Information: Forgotten Variable in Physics Models

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Abstract: The paper discusses a possible expansion of modern physics to include physics of Life since all attempts to create livings from non-living matter failed. Prior to the discussion, a life/non-life criterion is proposed: unlike physical systems, Livings can move from disorder to order without an external interference if the model of life concentrates upon the concept of **intelligent behavior** and does not include such "auxiliary" processes as metabolism and reproduction. A **mathematical formalism** suggests that a hypothetical "particle of life" (L-particle) can be represented by a quantum-classical hybrid in which the force following from the quantum potential is replaced by the **information** force. Besides the capability to move against the second law of thermodynamics, L-particle acquires such properties like self-image, self-awareness, self-supervision, est. that are typical for Livings. However since the L-particle being a quantum-classical hybrid acquires non-Newtonian and non-quantum properties, it does not belong to the physics matter as we know it: the modern physics should be complemented with the concept of the *information force* that represents a bridge between non-living and living matter. It has been suggested that quantum mechanics and mechanics of livings are sub-classes of a broader physical model, and the difference between them is due to different Liouville feedbacks, i.e. in different information forces. At this stage, L-particle is introduced as an **abstract mathematical concept** that is satisfied only to mathematical rules and assumptions, and its physical representation is still an open problem.

1. Introduction

The discovery of the Higgs boson and the following from it completeness of the physical picture of our Universe raised many questions, and one of them is the ability to create Life and Mind out of physical matter without any additional entities. The main differ-

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ence between living and non-living matter is in directions of their evolution: it has been recently recognized that the evolution of livings is progressive in a sense that it is directed to the highest levels of complexity if the complexity is measured by an irreducible number of different parts that interact in a well-regulated fashion. Such a property is not consistent with the behavior of *isolated* Newtonian systems that cannot increase their complexity without external forces. That difference created so called Schrödinger paradox: in a world governed by the second law of thermodynamics, all isolated systems are expected to approach a state of maximum *disorder*; since life approaches and maintains a highly *ordered* state – one can argue that this violates the Second Law implicating a paradox,[1]. But livings are not isolated due to such processes as metabolism and reproduction: the increase of order inside an organism is compensated by an increase in disorder outside this organism, and that removes the paradox. Nevertheless it is still tempting to find a mechanism that drives livings from disorder to the order. The purpose of this paper is to demonstrate that moving from a disorder to the order is not a prerogative of open systems: an isolated system can do it without help from outside. However such system cannot belong to the world of the modern physics: it belongs to the world of living matter, and that lead us to a concept of a living particle (L-particle) that is supposed to complement the modern physics. In order to introduce such a particle, we start with an idealized mathematical model of livings by addressing only one aspect of Life: a ***biosignature***, i.e. ***mechanical*** invariants of Life, and in particular, the ***geometry and kinematics of intelligent behavior*** of Livings disregarding other aspects of Life such as metabolism and reproduction. By narrowing the problem in this way, we are able to extend the mathematical formalism of physics' First Principles to include description of intelligent behavior of Livings. At the same time, by ignoring metabolism and reproduction, we can make the system isolated, and it will be a challenge to show that it still can move from a disorder to the order.

It seems unreasonable to introduce completely new principles for Living's behavior since they must obey the First Principles of Newtonian mechanics, although these Principles are necessary, but not sufficient: they should be complemented by additional statements linked to the Second Law of thermodynamics.

2.   Dynamical Model of Hypothetical L-particle

Let us start with the simplest dynamical equation of one-dimensional motion of a particle of unit mass under action of a force f depending on velocity v and time t

$$\dot{v} = f(v, t), \tag{1}$$

If initial conditions are not deterministic, and their probability density is given in the form

$$\rho_0 = \rho_0(V), \text{ where } \rho \geq 0, \text{ and } \int_{-\infty}^{\infty} \rho dV = 1 \tag{2}$$

while ρ is a single- valued function, then the evolution of this density is expressed by the corresponding Liouville equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial V}(\rho f) = 0 \tag{3}$$

The solution of this equation subject to initial conditions and normalization constraint (2) determines probability density as a function of V and t

$$\rho = \rho(V, t) \tag{4}$$

Remark. *Theory of stochastic differential equations makes distinction between the random variable v(t) and its values V in probability space, and we will adopt this rule.*

So far we are dealing with the Newtonian dynamics.

Let us now specify the force f as a feedback from the Liouville equation

$$f(v, t) = \varphi[\rho(v, t)] \tag{5}$$

and analyze the motion after substituting the force (5) into Eq.(1)

$$\dot{v} = \varphi[\rho(v, t)], \tag{6}$$

This is a fundamental step in our approach. Although the theory of ODE *does not* impose any restrictions upon the force as a function of space coordinates, the Newtonian physics *does*: equations of motion are never coupled with the corresponding Liouville equation. Nevertheless such a Liouville feedback does not lead to any physical inconsistence. As a matter of fact, quantum mechanics is based upon similar feedback in the form of the quantum potential. Indeed for a particle of mass m in a potential F, the Madelung version of the Schrödinger equation takes the following form

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\frac{\rho}{m}\nabla S) = 0 \tag{7}$$

$$\frac{\partial S}{\partial t} + (\nabla S)^2 + F - \frac{\hbar^2 \nabla^2 \sqrt{\rho}}{2m\sqrt{\rho}} = 0 \tag{8}$$

Here ρ and S are the components of the wave function $\psi = \sqrt{\rho}e^{iS/\hbar}$, and $\hbar$ is the Planck constant divided by 2π. The last term in Eq. (8) is known as quantum potential. From the viewpoint of Newtonian mechanics, Eq. (7) is the Liouville equation that expresses continuity of the flow of probability density, and Eq. (8) is the Hamilton-Jacobi equation for the action S of the particle. Actually the quantum potential in Eq. (8), as a feedback from Eq. (7) to Eq. (8), represents the difference between the Newtonian and quantum mechanics, and therefore, it is solely responsible for fundamental quantum properties. Strictly speaking, the quantum potential being uniquely defined by the probability density and its space derivatives can be associated with a specially selected *information force*. Recall that information is an indirectly observed quantity that is defined via entropy as

a measure of unpredictability: For a random variable X with n outcomes, the Shannon information denoted by $H(X)$, is

$$H(X) = -\sum_{i=1}^{n} \rho(x_i) \log_b \rho(x_i)$$

In our further applications we will use the continuous version of this formula

$$H(X) = -\int_{-\infty}^{\infty} \rho(x) \ln \rho(x) dx \tag{9}$$

Actually our approach is based upon a modification of the Madelung equation, and in particular, upon replacing the quantum potential with a different Liouville feedback.

Returning to Eq. (5), let specify this feedback as

$$f = \frac{\zeta}{\rho(v,t)} \int_{-\infty}^{v} [\rho(\eta,t) - \rho^*(\eta)] d\eta \tag{10}$$

Here $\rho^*(v)$ is a preset probability density satisfying the constraints (2), and ζ is a positive constant with dimensionality [1/sec]. As follows from Eq. (9), f has dimensionality of a force per unit mass that depends upon the probability densityρ, and therefore, it can be associated with the concept of information, so we will call it the *information force*. In this context, the coefficient ζ can be associated with the Planck constant that relates Newtonian and *information forces*. But since we are planning to deal with livings that belong to the macro-world, ζ must be of order of a viscose friction coefficient.

With the feedback (10), Eqs. (1) and (3) take the form, respectively

$$\dot{v} = \frac{\zeta}{\rho(v,t)} \int_{-\infty}^{v} [\rho(\eta,t) - \rho^*(\eta)] d\eta \tag{11}$$

$$\frac{\partial \rho}{\partial t} + \zeta[\rho(t) - \rho^*] = 0 \tag{12}$$

The last equation has the analytical solution

$$\rho = [(\rho_0 - \rho^*)e^{-\zeta t} + \rho^*] \tag{13}$$

Subject to the initial condition

$$\rho(t=0) = \rho_0 \tag{14}$$

that satisfy the constraints (2).

This solution converges to a preset stationary distribution $\rho^*(V)$. Obviously the normalization condition for ρ is satisfied if it is satisfied for ρ_0 and $\rho*$. Indeed,

$$\int_{-\infty}^{\infty} \rho V dV = [\int (\rho_0 - \rho^*)V dV]e^{-\zeta t} + \int_{-\infty}^{\infty} \rho^* V dV = 1 \tag{15}$$

Rewriting Eq. (13) in the form

$$\rho = \rho_0 + \rho^*(1 - e^{-\zeta t}) \tag{16}$$

one observes that $\rho \geq 0$ at all $t \geq 0$ and $-\infty > V > \infty$.

As follows from Eq. (13), the solution of Eq. (12) has an attractor that is represented by the preset probability density $\rho^*(v)$. Substituting the solution (13) in to Eq. (11), one arrives at the ODE that simulates the stochastic process with the probability distribution (13)

$$\dot{v} = \frac{\zeta e^{-\zeta t}}{[\rho_0(v) - \rho^*(v)]e^{-\zeta t} + \rho^*(v)} \int\limits_{-\infty}^{v} [\rho_0(\eta) - \rho^*(\eta)]d\eta \tag{17}$$

As will be shown below, the randomness of the solution of Eq. (6) is caused by instability that is controlled by the corresponding Liouville equation.

It is reasonable to assume that the solution (13) starts with a sharp initial condition

$$\rho_0(V) = \delta(V) \tag{18}$$

As a result of that assumption, all the randomness is supposed to be generated *only* by the controlled instability of Eq. (16). Substitution of Eq. (17) into Eq. (16) leads to two different domains of v:

$v \neq 0$

and

$v = 0$

where the solution has two different forms, respectively

$$\int\limits_{-\infty}^{v} \rho^*(\zeta)d\zeta = \left(\frac{C}{e^{-\zeta t} - 1}\right)^{1/\zeta}, v \neq 0 \tag{19}$$

$$v \equiv 0 \tag{20}$$

Indeed, $\dot{v} = \frac{\zeta e^{-\zeta t}}{\rho^*(v)(e^{-\zeta t}-1)} \int\limits_{-\infty}^{v} \rho^*(\eta)]d\eta$

whence $\frac{\rho*(v)}{\int\limits_{-\infty}^{v} \rho*(\eta)d\eta}dv = \frac{\zeta e^{-\zeta t}}{e^{-\zeta t}-1}dt$. Therefore, $\ln \int\limits_{-\infty}^{v} \rho*(\eta)d\eta = \ln(\frac{C}{e^{-\zeta t}-1})^{1/\zeta}$ and that leads to Eq. (19) that presents an implicit expression for v as a function of time since ρ^* is the known function. Eq. (20) represents a singular solution, while Eq. (19) is a regular solution that include arbitrary constant C. The regular solutions is unstable at $t = 0, |v| \to 0$ where the Lipschitz condition is violated

$$\frac{d\dot{v}}{dv} \to \infty \text{ at } t \to 0, |v| \to 0 \tag{21}$$

and therefore, an initial error always grows generating *randomness*.

Let us analyze the behavior of the solution (19) in more details. As follows from this solution, all the particular solutions for different values of C intersect at the same

point $v = 0$ at $t = 0$, and that leads to non-uniqueness of the solution due to violation of the Lipcshitz condition. Therefore, the same initial condition$v = 0$ at $t = 0$ yields infinite number of different solutions forming a family (19); each solution of this family appears with a certain probability guided by the corresponding Liouville equation (12). For instance, in cases plotted in Fig.1 a and Fig.1 b, the "winner" solution is, respectively,
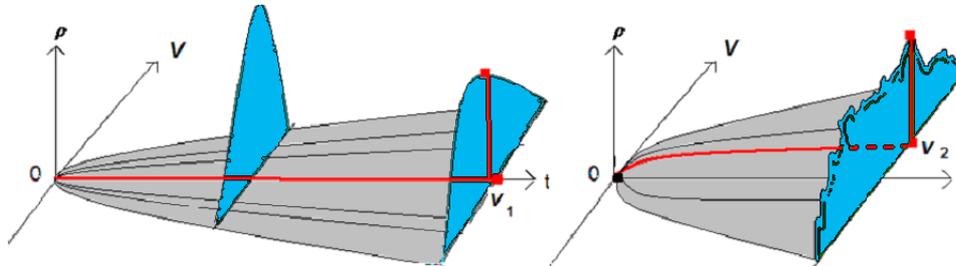

Fig. 1 Stochastic processes and their attractors

$$v_1 = \varepsilon \to 0, \quad \rho(v_1) = \rho_{\max}, \text{ and } v = v_2, \quad \rho(v_2) = \sup\{\rho\} \qquad (22)$$

since it passes through the maximum of the probability density . However, with lower probabilities, other solutions of the same family can appear as well. Obviously, this is a non-classical effect. Qualitatively, this property is similar to those of quantum mechanics: the system keeps all the solutions simultaneously and displays each of them "by a chance", while that chance is controlled by the evolution of probability density (13).

Let us emphasize the connections between solutions of Eqs. (11) and (12): the solution of Eq. (11) is an one-parametrical family of trajectories (19), and each trajectory occurs with the probability described by the solution (16) of Eq. (12). But since the probability (16) depends upon time, an L-particle cannot stay on the same fixed (random) trajectory: as a quantum particle, it is capable to jump to any other trajectory of the family, and therefore, it "simultaneously" occupies all the space available.

The approach is generalized to n-dimensional case simply by replacing v with a vector $v = v_1, v_2, ...v_n$ since Eq. (12) does not include space derivatives

$$\dot{v}_i = \frac{\zeta}{\rho(\{v\}, t)} \int\limits_{-\infty}^{v_i} [\rho(\{\eta\}, t) - \rho^*(\{\eta\})] d\eta_i \qquad (23)$$

$$\frac{\partial \rho(\{V\}, t)}{\partial t} + n\zeta\rho(\{V\}t) - \rho^*(\{V\}) = 0 \qquad (24)$$

Examples. Let us start with the following normal distribution

$$\rho^*(V) = \frac{1}{\sqrt{2\pi}} e^{-\frac{V^2}{2}} \qquad (25)$$

Substituting the expression (23) and (17) into Eq. (18) at $X = x$, and $\zeta = 1$ one obtains

$$v = erf^{-1}(\frac{C_1}{e^{-t} - 1}), v \neq 0 \qquad (26)$$

As another example, let us choose the target density ρ^* as the Student's distribution, or so-called power law distribution

$$\rho^*(V) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})}(1 + \frac{V^2}{\nu})^{-(\nu+1)/2} \qquad (27)$$

Substituting the expression (24) into Eq. (18) at $V = v$, $\nu=1$, and $\zeta = 1$ one obtains

$$v = \cot(\frac{C}{e^{-t} - 1}) \text{ for } v \neq 0 \qquad (28)$$

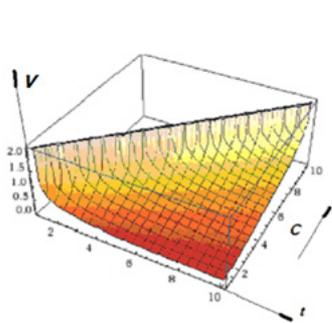The 3D plot of the solutions of Eqs.(26) and (28), are presented in Figures 2, and 3, respectively.

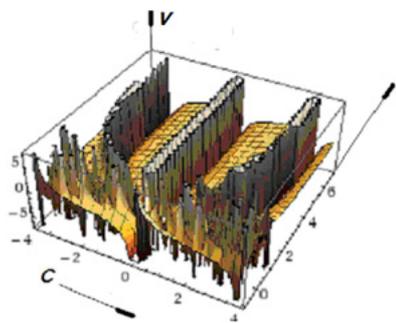**Fig. 2** Dynamics driving random events to normal distribution.

Fig. 3 Dynamics driving random events to power law distribution.

3. Departure of L-particle From Newtonian Dynamics

We will start with mathematical formulation of n-dimensional versions of L-particle Eqs. (23) and (24).

The model is represented by a system of nonlinear ODE (Eqs.23), and a linear ODE (Eq. (24) coupled in a master-slave fashion: Eq. (24), is to be solved independently, prior to solving Eqs.(23).

The terms describing the contributions of the feedbacks in Eqs. (23), have the dimensionality of acceleration, and therefore, they can be interpreted as forces referred to unit mass. In order to distinguish them from "physical" forces that are associated with energy and that are not included into the equations of motion, we will call them ***information*** forces.

The major departure of mathematical property of L-particle from that of Newtonian systems is in transition from the determinism to randomness and formation of stochastic attractors. Unlike chaos in Newtonian systems, randomness in L-particle dynamics is fully controlled by the Liouville equation (24) via the information force Eq. (10). The

Langeven version of Newtonian dynamics has similar dynamical structure to that of L-particle, however the fundamental difference between them is in the origin of randomness: in Langeven systems, randomness is due to random external input, while in L-particle it is self-generated. In terms of transition to randomness, this departure is toward quantum mechanics, and that could be expected based upon similarity between dynamical structures of L-particle and quantum systems, (see Fig. 4): in both cases there is a feedback from the Liouville equation to the equation of motion although these feedback are different. Indeed, the Madelung version of the Schrödinger equation has the

feedback from the Liouville equation to the Hamilton-Jacobi equation in the form of quantum potential, (see Eqs. (7) and (8)) , while in L-particle the feedback is represented by the information force (10). Another fundamental departure from Newtonian dynamics is a violation of the second law of thermodynamics by the capability of moving from disorder to order without a hepp from outside. In order to demonstrate it, let us turn to Eq. (13) and select the initial ρ_0 and the preset (target) ρ^* densities such that

$$H(\rho_0) = -\int\limits_{-\infty}^{\infty} \rho_0 \ln \rho_0 dV > H(\rho^*) = -\int\limits_{-\infty}^{\infty} \rho^* \ln \rho^* dV \qquad (29)$$

Then expressing Eq. (8) in terms of entropy, one obtains

$$H(\rho) = [H(\rho_0) - H(\rho^*)]e^{-t} + H(\rho^*) \qquad (30)$$

whence

$$\frac{dH(\rho)}{dt} < 0 \qquad (31)$$

i.e. the entropy of the current probability density is monotonously decreasing. Therefore, L-particle with is not necessarily complies with the second law of thermodynamics.

It should be emphasized that if the feedback (10) is de-activated, the L-particle becomes a simple Newtonian one.

4.   Similarity of L-particle and Quantum Mechanics

In this section we will demonstrate that L-particle is similar, but not identical to a quantum particle. Such a conclusion should be expected since the dynamical structure of L-particle differs from a quantum particle only by the type of the feedback from the Liouville equation, (see Fig. 4).

α.*Superposition.* In quantum mechanics, any observable quantity corresponds to an eigenstate of a Hermitian linear operator. The linear combination of two or more eigenstates results in quantum superposition of two or more values of the quantity. If the quantity is measured, the projection postulate states that the state will be randomly collapsed onto one of the values in the superposition (with a probability proportional to the square of the amplitude of that eigenstate in the linear combination). Let us compare the behavior of the L-particle from that viewpoint. As follows from Eq. (19), all the particular solutions intersect at the same point $v = 0$ at $t = 0$, and that leads to non-uniqueness

of the solution due to violation of the Lipcshitz condition (see Eq. (21)). Therefore, the same initial condition$v = 0$ at $t = 0$yields infinite number of different solutions forming a family (19); each solution of this family appears with a certain probability guided by the Liouville equation (12). For instance, in case of Eq. (25), the "winner" solution is $v \equiv 0$ since it passes through the maxima of the probability density. However, with lower probabilities, other solutions of the family (26) can appear as well. Obviously, this is a non-classical effect. Qualitatively, this property is similar to those of quantum mechanics: the system keeps all the solutions simultaneously and displays each of them "by a chance", while that chance is controlled by the evolution of probability density (12). As noticed in the Section 2, since the probability (16) depends upon time, an L-particle cannot stay on the same fixed (random) trajectory: as a quantum particle, it is capable to jump to any other trajectory of the family, and therefore, it "simultaneously" occupies all the space available.
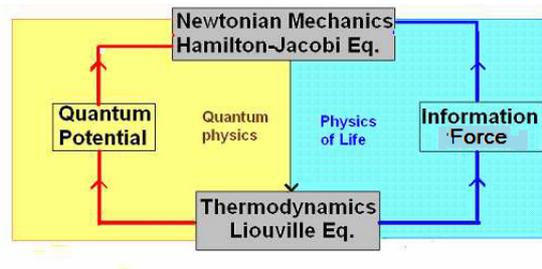

Fig. 4 Topological similarity between Quantum Mechanics and L-particle

*b. Uncertainty Principle.*In quantum physics, the Heisenberg uncertainty principle states that one cannot measure values (with arbitrary precision) of certain conjugate quantities, which are pairs of observables of a single elementary particle. These pairs include the position and momentum. Similar (but not identical) relationship follows from Eq. (19) after differentiating it over time and elimination t

$$\dot{v}\rho^*(v)[\int_{-\infty}^{v} \rho^*(\zeta)d\zeta]^2 + [\int_{-\infty}^{v} \rho^*(\zeta)d\zeta]^{-1} = Const. \tag{32}$$

i.e. the combination (32) of the position and the velocity of L- particle is constant along a fixed trajectory. In particular, at $t = 0$, v and $\dot{v}$cannot be defined separately.

c. Wave–particle duality. In physics, wave–particle duality is a conceptualization that all objects in our universe exhibit properties of both waves (such as non-locality) and of particles (such as quantization of some of their properties). As shown by Max Born, the wave associated with the electron is not a tangible 'matter wave', but one that determines the probability of scattering of the electron in different directions. Similar "duality" follows from the hybrid model. Indeed, Eq. (11) describes the trajectories of particles, while Eq. (12) represents the wave of probability that captures the particle "scattering". Thus, the similarity between the L-particle and quantum systems is due to a feedback from the Liouville equation to the equation of motion (that does not exist in

Newtonian physics), while the difference between these models is due to different types of feedbacks, i.e. between quantum potential and information force.

   d. Entanglement. Quantum entanglement is a phenomenon in which the quantum states of two or more objects have to be described with reference to each other, even though the individual objects may be spatially separated. Qualitatively similar effect in L-particles will be demonstrated below. Let us consider a two dimensional case of L-particle follows from Eqs. (23) and (24) for $\zeta = 1$

$$\dot{v}_1 = \frac{1}{2\rho(v_1, v_2, t)} \int_{-\infty}^{v_1} [\rho(\eta, v_2, t) - \rho^*(\eta, v_2)]d\eta \tag{33}$$

$$\dot{v}_2 = \frac{1}{2\rho(v_1, v_2, t)} \int_{-\infty}^{v_2} [\rho(v_1, \eta, t) - \rho^*(v_1, \eta)]d\eta \tag{34}$$

$$\frac{\partial \rho(V, t)}{\partial t} + \rho(V, t) - \rho^*(V) = 0 \tag{35}$$

The solution of Eq. (35) has the same form as for one-dimensional case, (see Eq. (16)

$$\rho = \rho_0 + \rho^*(1 - e^{-t}) \tag{36}$$

Substitution this solution into Eqs. (33) and (34), yields, respectively

$$\dot{v}_1 = \frac{e^{-t}}{2[\rho_0(v_1, v_2) - \rho^*(v_1, v_2)]e^{-t} + \rho^*(v_1, v_2)} \int_{-\infty}^{v_1} [\rho_0(\eta, v_2) - \rho^*(\eta, v_2)]d\eta \tag{37}$$

$$\dot{v}_2 = \frac{e^{-t}}{2[\rho_0(v_1, v_2) - \rho^*(v_1, v_2)]e^{-t} + \rho^*(v_1, v_2)} \int_{-\infty}^{v_2} [\rho_0(v_1, \eta) - \rho^*(v_1, \eta)]d\eta \tag{38}$$

that are similar to Eq. (17). Following the same steps as in one-dimensional case, one arrives at the following solutions of Eqs. (37) and (38) respectively

$$\int_{-\infty}^{v_1} \rho^*(\eta, v_2)d\eta = \frac{C_1}{e^{-t} - 1}, v_1 \neq 0 \tag{39}$$

$$\int_{-\infty}^{v_2} \rho^*(\eta, v_1)d\eta = \frac{C_2}{e^{-t} - 1}, v_2 \neq 0 \tag{40}$$

that are similar to the solution (19). Since $\rho(v_1, v_2)$ is the known (preset) function, Eqs. (39) and (40) implicitly define v_1 and v_2 as functions of time. Elimination time t and orbitary constants C_1, C_2, on obtains

$$\frac{d}{dt}[\ln \int_{-\infty}^{v_1} \rho^*(\eta, v_2)d\eta]/\frac{d}{dt}[\ln \int_{-\infty}^{v_2} \rho^*(v_1, \eta)d\eta] \equiv 1 \tag{41}$$

Thus, the ratio (41) is deterministic although both the numerator and denominator are random,(see Eqs.(39) and 940)). This is a fundamental non-classical effect representing a global constraint. Indeed, in theory of stochastic processes, two random functions are considered statistically equal if they have the same statistical invariants, but their point-to-point equalities are not required (although it can happen with a vanishingly small probability). As demonstrated above, the *diversion of determinism into randomness via instability (due to a Liouville feedback), and then conversion of randomness to partial determinism (or coordinated randomness) via entanglement* is the fundamental non-classical paradigm.

Let us discuss more general characteristic of entanglement in physics.

a. Criteria for non-local interactions. Based upon analysis of all the known interactions in the Universe and defining them as local, one can formulate the following criteria of *non*-local interactions: they are *not* mediated by another entity, such as a particle or field; their actions are *not* limited by the speed of light; the strength of the interactions does not drop off with distance. All of these criteria lead us to the concept of the global constraint as a starting point.

b. Global constraints in physics. It should be recalled that the concept of a global constraint is one of the main attribute of Newtonian mechanics. It includes such *idealizations* as a rigid body, an incompressible fluid, an inextensible string and a membrane, a non-slip rolling of a rigid ball over a rigid body, etc. All of those idealizations introduce geometrical or kinematical restrictions to positions or velocities of particles and provides "instantaneous" speed of propagation of disturbances. Let us discuss the role of the reactions of these constraints. One should recall that in an incompressible fluid, the reaction of the global constraint $\nabla \cdot v \geq 0$ (expressing non-negative divergence of the velocity v) is a non-negative pressure $p \geq 0$; in inextensible flexible (one- or two-dimensional) bodies, the reaction of the global constraint $g_{ij} \leq g_{ij}^0, i, j = 1, 2$ (expressing that the components of the metric tensor cannot exceed their initial values) is a non-negative stress tensor $\sigma_{ij} \geq 0, i, j = 1, 2$. It should be noticed that all the known forces in physics (the gravitational, the electromagnetic, the strong and the weak nuclear forces) are local. However, the reactions of the global constraints listed above do not belong to any of these local forces, and therefore, they are non-local. Although these reactions are being successfully applied for engineering approximations of theoretical physics, one cannot relate them to the origin of **entanglement** since they are result of idealization that ignores the discrete nature of the matter. However, there is another type of the global constraint in physics: the normalization constraint, (see Eq. (2). This constraint is fundamentally different from those listed above for two reasons. Firstly, it is not an idealization, and therefore, it cannot be removed by taking into account more subtle properties of matter such as elasticity, compressibility, discrete structure, etc. Secondly, it imposes restrictions not upon positions or velocities of particles, but upon the *probabilities* of their positions or velocities, and that is where the **entanglement** comes from. Indeed, if the Liouville equation is coupled with equations of motion as in quantum mechanics, the normalization condition imposes a global constraint upon the state variables, and that is the origin of quantum

entanglement. In quantum physics, the reactions of the normalization constraints can be associated with the energy eigenvalues that play the role of the Lagrange multipliers in the conditional extremum formulation of the Schrödinger equation, [2]. In L-particle model, the Liouville equation is also coupled with equations of motion (although the feedback is different). And that is why the origin of entanglement in L-particle model is the same as in quantum mechanics.

c. Speed of action propagation. Further illumination of the concept of quantum entanglement follows from comparison of quantum and Newtonian systems. Such a comparison is convenient to perform in terms of the Madelung version of the Schrödinger equation, see Eqs. (7) and (8). As follows from these equations, the Newtonian mechanics ($\hbar = 0$), in terms of the S and ρ as state variables, is of a hyperbolic type, and therefore, any discontinuity propagates with the *finite* speed S/m, i.e. the Newtonian systems do not have non-localities. But the quantum mechanics ($\hbar \neq 0$) is of a parabolic type. This means that any disturbance of S or ρ in one point of space instantaneously transmitted to the whole space, and this is the mathematical origin of non-locality. But is this a unique property of quantum evolution? Obviously, it is not. Any parabolic equation (such as Navier-Stokes equations or Fokker-Planck equation) has exactly the same non-local properties. However, the difference between the quantum and classical non-localities is in their physical interpretation. Indeed, the Navier-Stokes equations are derived from simple laws of Newtonian mechanics, and that is why a physical interpretation of non-locality is very simple: If a fluid is incompressible, then the pressure plays the role of a reaction to the geometrical constraint $\nabla \cdot v \geq 0$, and it is transmitted instantaneously from one point to the whole space (the Pascal law). One can argue that the incompressible fluid is an idealization, and that is true. However, it does not change our point: Such a model has a lot of engineering applications, and its non-locality is well understood. The situation is different in quantum mechanics since the Schrodinger equation has never been derived from Newtonian mechanics: It has been postulated. In addition to that, the solutions of the Schrodinger equation are random, while the ***origin*** of the randomness does not follow from the Schrodinger formalism. That is why the physical origin of the same mathematical phenomenon cannot be reduced to simpler concepts such as "forces": It should be accepted as an attribute of the Schrodinger equation.

Let us turn now to the L-particle model. The formal difference between them and quantum systems is in a feedback from the Liouville equation to equations of motion: the gradient of the quantum potential is replaced by the information forces, while the equations of motion are written in the form of the second Newton's law rather than in the Hamilton-Jacoby form. The corresponding Liouville equation becomes ODE (see Eq. (12)) where the state variables play the role of parameters; it can be easily seen from the solution (13) that all the changes in the state variables occurs simultaneously. Thus, both quantum systems and L-particle model possess the same non-locality: instantaneous propagation of changes in the probability density, and this is due to similar topology of their dynamical structure, and in particular, due to a feedback from the Liouville equation.

d. Origin of randomness in physics. Since entanglement in quantum systems as well as in L-particle model are exposed via instantaneous propagation of changes in the probability density, it is relevant to ask what the origin of randomness in physics is. The concept of randomness has a long history. Its philosophical aspects first were raised by Aristotle, while the mathematical foundations were introduced and discussed much later by Henry Poincare who wrote: "A very slight cause, which escapes us, determines a considerable effect which we cannot help seeing, and then we say this effect is due to chance". Actually Poincare suggested that the origin of randomness in physics is the dynamical instability, and this viewpoint has been corroborated by theory of turbulence and chaos. However, the theory of dynamical stability developed by Poincare and Lyapunov revealed the main flaw of physics: its fundamental laws do not discriminate between stable and unstable motions. But unstable motions cannot be realized and observed, and therefore, a special mathematical analysis must be added to found out the existence and observability of the motion under consideration. However, then another question can be raised: why turbulence as a post-instability version of an underlying laminar flow can be observed and measured? In order to answer this question, we have to notice that the concept of stability is an attribute of mathematics rather than physics, and in mathematical formalism, stability must be referred to the corresponding class of functions. For example: a laminar motion with sub-critical Reynolds number is *stable* in the class of deterministic functions. Similarly, a turbulent motion is *stable* in the class of random functions. Thus the same physical phenomenon can be unstable in one class of functions, but stable in another, enlarged class of functions.

Thus, we are ready now to the following conclusion: any stochastic process in *Newtonian* dynamics describes the physical phenomenon that is unstable in the class of the deterministic functions.

This elegant union of physics and mathematics has been disturbed by the discovery of quantum mechanics that complicated the situation: Quantum physicists claim that quantum randomness is the "true" randomness unlike the "deterministic" randomness of chaos and turbulence. Richard Feynman in his "Lectures on Physics" stated that randomness in quantum mechanics in *postulated*, and that closes any discussions about its origin. However, recent result disproved existence of the "true" randomness. Indeed, as shown in Zak, M., 2009, the origin of randomness in quantum mechanics can be traced down to *instability* generated by quantum potential at the point of departure from a deterministic state if for dynamical analysis one transfer from the Shrodinger to the Madelung equation. (For details see Section I.4.5**).** As demonstrated there, the instability triggered by failure of the Lipchitz condition splits the solution into a continuous set of random samples representing a "bridge" to quantum world. Hence, now we can state that any stochastic process in physics describes the physical phenomenon that is unstable in the class of the deterministic functions. Actually this statement can be used as a definition of randomness in physics.

5. Difference Between L-particle and Quantum Systems

Despite several similarities, the L-particle model is fundamentally different from quantum systems: its dynamics is *not* conservative. That is why the quantum potential is replaced by information forces rather than information potential: such a potential does not exist. For the same reason, the Hamilton-Jacobi equations are replaced by the second Newton's law. As a consequence of that difference, the unitary evolution has been lost.

6. Biological Interpretation of L-particle.

a.Biological viewpoint.   Actually the model under discussion was inspired by E. Schrödinger, the creator of quantum mechanics who wrote in his book "What is Life": "Life is to create order in the disordered environment against the second law of thermodynamics",[1]. The proposed model illuminates the "border line" between living and non-living systems. The model introduces a L-particle that, in addition to Newtonian properties, possesses the ability to process information. The probability density can be associated with the *self-image* of the L-particle as a member of the class to which this particle belongs, while its ability to convert the density into the information force - with the *self-awareness* (both these concepts are adopted from psychology). Continuing this line of associations, the equation of motion (see Eq. (11)) can be identified with a motor dynamics, while the evolution of density (see Eq.. (12)) –with a mental dynamics. Actually the mental dynamics plays the role of the Maxwell sorting demon: it rearranges the probability distribution by creating the information force and converting it into a force that is applied to the particle. One should notice that mental dynamics describes evolution of the whole class of state variables (differed from each other only by initial conditions), and that can be associated with the ability to generalize that is a privilege of living systems. Continuing our biologically inspired interpretation, it should be recalled that the second law of thermodynamics states that the entropy of an isolated system can only increase. This law has a clear probabilistic interpretation: increase of entropy corresponds to the passage of the system from less probable to more probable states, while the highest probability of the most disordered state (that is the state with the highest entropy) follows from a simple combinatorial analysis. However, this statement is correct only if there is no Maxwell' sorting demon, i.e., nobody inside the system is rearranging the probability distributions. But this is precisely what the Liouville feedback is doing: it takes the probability density ρ from Equation (12), creates functions of this density, converts them into a force and applies this force to the equation of motion (1). As demonstrated by Eq.(30), the evolution of the probability density may lead to the entropy decrease "against the second law of thermodynamics".

Obviously the last statement should not be taken literary; indeed, the proposed model captures only those aspects of the living systems that are associated with their behavior, and in particular, with their motor-mental dynamics, since other properties are beyond the dynamical formalism. Therefore, such physiological processes that are needed for

the metabolism are not included into the model. That is why this model is in a formal disagreement with the second laws of thermodynamics while the living systems are not. Indeed, applying the second law of thermodynamics, we consider our system as isolated one while the underlying real system is open due to other activities of livings that were not included in our model. Nevertheless, despite these limitations, the L-particle model capture the "magic" of Life: the ability to create self-image and self-awareness and move from disorder to the order.

   b. *Psychological viewpoint.* The proposed model can be interpreted as representing interactions of the L- particle, or living agent with the self-image and the images of other agents via the mechanisms of self-awareness. In order to associate these basic concepts of psychology with our mathematical formalism, we have to recall that living systems can be studied in many different spaces such as physical (or geographical) space as well as abstract (or conceptual) spaces. The latter category includes, for instance, social class space, sociometric space, social distance space, semantic space est. Turning to our model, one can identify two spaces: the physical space v, t in which the agent state variables $v_i = \dot{x}_i$ evolve,(see Eqs.(1)), and an abstract space in which the probability density of the agent' state variables evolve (see Eq.(3)).The connection with these spaces have been already described earlier: if Eqs. (1) are run many times starting with the same initial conditions, one will arrive at an ensemble of different random solutions, while Eq. (3) will show what is the probability for each of these solutions to appear. Thus, Eq. (3) describes the general picture of evolution of the communicating agents that does not depend upon particular initial conditions. Therefore, the solution of this equation can be interpreted as the evolution of the self- and non-self images of the agents that jointly constitutes the *collective mind* in the probability space, Fig. (5).
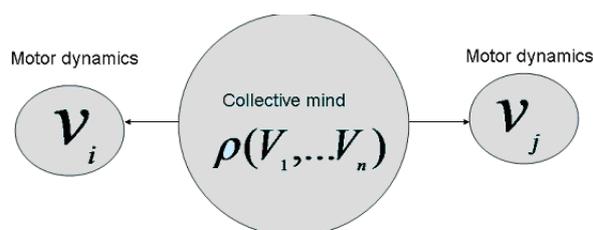

Fig. 5 Collective mind.

   Based upon that, one can propose the following interpretation of the model of communicating agents: considering the agents as L-particles, one can identify Eqs. (1) as a model simulating their motor dynamics, i.e. actual motions in physical space, while Eq.(3) as the *collective mind* composed of mental dynamics of the agents. Such an interpretation is evoked by the concept of reflection in psychology. Reflection is traditionally understood as the human ability to take the position of an observer in relation to one's own thoughts. In other words, the reflection is the *self-awareness* via the interaction with the image of the self. Hence, in terms of the phenomenological formalism proposed above, a non-living system may possess the self-image, but it is not equipped with the self-awareness, and therefore, this self-image is not in use. On the contrary, in living

systems the self-awareness is represented by the information forces that send information from the self-image (3) to the motor dynamics (1). Due to this property that is well-pronounced in the proposed model, a living agent can run its mental dynamics ahead of real time, (since the mental dynamics is fully deterministic, and it does not depend explicitly upon the motor dynamics) and thereby, it can predict future expected values of its state variables; then, by interacting with the self-image via the information forces, it can change the expectations if they are not consistent with the objective. Such a self-supervised dynamics provides a major advantage for the corresponding living agents, and especially, for biological species: due to the ability to predict future, they are better equipped for dealing with uncertainties, and that improves their survivability. It should be emphasized that the proposed model, strictly speaking, does not discriminate living systems of different kind in a sense that all of them are characterized by a self-awareness-based feedback from mental (3) to motor (1) dynamics. However, in primitive living systems (such as bacteria or viruses) the self-awareness is reduced to the simplest form that is the self/no-self discrimination; in other words, the difference between the living systems is represented by the level of complexity of that feedback.

7.   L-particle's Intelligence

A human intelligence has always been a mystery for physicists and an obstacle for artificial intelligence. It was well understood that human behavior and, in particular, the decision making process are governed by feedbacks from the external world, and this part of the problem was successfully simulated in the most sophisticated way by control systems. However, in addition to that, when the external world does not provide sufficient information, a human turns for "advice" to his experience and that is associated with a human intelligence. In L-particle, inelligent behavior is captured by the system of random ODE, and the "advice" is implemented by a feedback from the Liouville equation ***generated by the original system itself***. Physical intelligence can be defined as the ability of a system to move from disorder to order autonomously. Remaining within the framework of dynamical formalism discussed above, this property can be considered as the criterion of physical intelligence, and it can be formulated in terms of the rate of decrease of entropy $\frac{dH}{dt} < 0$. Then the absolute value of the rate of decrease of entropy $|\frac{dH}{dt}|$ can be chosen as a measure of physical intelligence. The objective of intelligent systems may not be linked to survivability. A more reasonable way is to define the objective of an intelligent system via optimization of an appropriately selected functional adopting the strategy of optimal control theory.

   a.   Optimization- is one of the privileges of Livings associated with intelligence. We will demonstrate below the capability of L-particle to perform optimization ***autonomously***. For that purpose, let us assume that for a survival the L-particle must find an optimal speed v_0 that deliver the global maximum to the function $\rho^*(v)$. Then turning to Eq. (11) and (12), one can see that at each fixed time the larger value of this function will have the higher probability to appear. Obviously with the lower probability other

values can appear as well. But usually nature does not provide the best solution: it gives us a solution that is "good enough". As follows from Eqs. (23) and (24), n L-particles perform collective optimization by maximizing an n-dimensional function $\rho(\{v\})$ via collective cooperation. It should be pointed out that the entropy of the system during the process of optimization could decrease or increase depending upon the initial state of the system, while the optimization process remains **autonomous.**

b. Prediction of future. Another evidence of L-particle's intelligence is a capability to **predict future** in terms of probability. In order to demonstrate it, let us turn to Eqs. (11) and (13). Since the mental dynamics (13) does not depend upon the motor dynamics (11), the L-particle could run its mental dynamics ahead of the initial time t_0 and find all the probability moments (mean, variance... est.) for any $t_¿$ t_0 thereby predicting future scenario in terms of probability. The situation is similar in n-dimensional case: as follows from Eqs. (23) and (24), each L-particle could predict not only its own future, but the future of any other L-particle as well.

c. Cooperation. Continuing the discussion of interaction between two L-particles, we start with their cooperation when these particles have the *same* objective that is: to approach a preset attractor, or to maximize a preset function. It is reasonable to assume that in this case their joint probability exists and is described by Eq. (35). We will assume that the particles do not know the values of state variables of each other and predict these values with help of their means found from the mental dynamics. However we will assume that the first L-particle admits that the second L-particle may know the state variable of the first one. The same is true for the second L-particle. Then instead of the systems (33), (34), we have the governing equation of the first L-particle

$$\dot{v}_1 = \frac{1}{2\rho(v_1, \bar{V}_2, t)} \int\limits_{-\infty}^{v_1} [\rho(\eta, \bar{V}_2, t) - \rho^*(\eta, \bar{V}_2)]d\eta \qquad (42)$$

where $\bar{V}_2$ is the expected value of V_2

$$\bar{V}_2 = \int\limits_{-\infty}^{\infty} V_2 \rho(V_1, V_2)dV_1 dV_2 \qquad (43)$$

and the governing equation of the *image* of the second L-particle in view of the first L-particle

$$\dot{v}_{2|1} = \frac{1}{2\rho(v_1, \bar{V}_2, t)} \int\limits_{-\infty}^{v_2} [\rho(v_1, \eta, t) - \rho^*(v_1, \eta)]d\eta \qquad (44)$$

where $v_{2|1}$ is the state variable of the second L-particle in view of the first L-particle.

Similar equations can be written for the second L-particle

$$\dot{v}_2 = \frac{1}{2\rho(\bar{V}_1, v_2, t)} \int\limits_{-\infty}^{v_1} [\rho(\bar{V}_1, \eta, t) - \rho^*(\bar{V}_1, \eta)]d\eta \qquad (45)$$

where $\bar{V}_1$ is the expected value of V_1

$$\bar{V}_1 = \int\limits_{-\infty}^{\infty} V_1 \rho(V_1, V_2) dV_1 dV_2 \tag{46}$$

and the governing equation of the *image* of the first L-particle in view of the second L-particle

$$\dot{v}_{1|2} = \frac{1}{2\rho(\bar{V}_1, v_2, t)} \int\limits_{-\infty}^{v_1} [\rho(\eta, v_2, t) - \rho^*(\eta, v_2)] d\eta \tag{47}$$

where $v_{1|2}$ is the state variable of the first L-particle in view of the second L-particle.

The corresponding Liouville equation that governs the joint probability equation is not changed: it is still given by Eq. (35). Its solution (36) should be substituted into Eqs. (42), (44) and Eqs. (45), (47) along with the Eqs. (43) and (46). After these substitutions, one arrives at the closed system. Actually there are two independent systems, and each of them describes *entanglement* of the L-particle with the image of the other L-particle. Each system has random solutions that appear with the probability described by Eq. (36).

Let us consider now the case when the L-particles do not know the values of state variables and do not even admit that each of them may know the state variable of another one.

Then instead of the systems (42),(44) and (45),(47) we have, respectively

$$\dot{v}_1 = \frac{1}{2\rho(v_1, \bar{V}_2, t)} \int\limits_{-\infty}^{v_1} [\rho(\eta_1, \bar{V}_2, t) - \rho^*(\eta_1, \bar{V}_2)] d\eta_1 \tag{48}$$

$$\dot{v}_{2|1|1} = \frac{1}{2\rho(\bar{V}_1, \bar{V}_2, t)} \int\limits_{-\infty}^{v_2} [\rho(\bar{V}_1, \eta_2, t) - \rho^*(\bar{V}_1, \eta_2)] d\eta_2 \tag{49}$$

$$\dot{v}_2 = \frac{1}{2\rho(\bar{V}_1, v_2, t)} \int\limits_{-\infty}^{v_1} [\rho(\bar{V}_1, \eta_2, t) - \rho^*(\bar{V}_1, \eta_2)] d\eta_2 \tag{50}$$

$$\dot{v}_{1||2|2} = \frac{1}{2\rho(\bar{V}_1, \bar{V}_2, t)} \int\limits_{-\infty}^{v_1} [\rho(\eta_1, \bar{V}_2, t) - \rho^*(\eta_1, \bar{V}_2)] d\eta_1 \tag{51}$$

Here $v_{2|1|1}$ is the state variable of the first L-particle view on the second one in view of the first one, and

$v_{1|2|2}$ is the state variable of the second L-particle view on the first one in view of the second one.

It is easy to conclude that the image equations (49) and (51) can be solved independently

$$v_{2|1|1} = \int\limits_0^t dt \frac{1}{2\rho(\bar{V}_1, \bar{V}_2, t)} \int\limits_{-\infty}^{v_2} [\rho(\bar{V}_1, \eta_2, t) - \rho^*(\bar{V}_1, \eta_2)] d\eta_2 \tag{52}$$

$$v_{1|2|2} = \int\limits_0^t dt \frac{1}{2\rho(\bar{V}_1, \bar{V}_2, t)} \int\limits_{-\infty}^{v_1} \rho(\eta_1, \bar{V}_2, t) - \rho^*(\eta_1, \bar{V}_2) d\eta_1 \tag{53}$$

Now replacing $\bar{V}_2, \bar{V}_1$ in Eqs.(48) and (50) by the solutions for $v_{2|1|1}$ and $v_{1|2|2}$, respectively, one arrives at two independent ODE describing behaviors of the players. Therefore, at this level of incompleteness of information, the entanglement disappears. More sofisticated reflection chains such as "What do you think I think you think..." have been discussed in [3] based upon the same mathematical formalism.

A cooperation with incomplete information give a reason to distinguish two type of dependence between the L-particles described by the variables v_i in L-particle systems. The first type of dependence is entanglement that has been introduced and discussed above. One should recall that in order to be entangled, the particles are supposed to run the system jointly during some initial period of time. But what happens if the particles had never been in contact? Obviously they are not entangled, i.e. they cannot predict each other motions. However they are not completely independent: they can make random decisions, but the probability of these decisions will be correlated via the joint probability. As a result, the particles will be able to predict ***expected*** decisions of each other. We will call such correlation a ***weak*** entanglement. As follows from the cooperation with incomplete information considered above, weak entanglement was presented as entanglement of a L-particle with the probabilistic image of another L-particle. More sophisticated cases of entanglement in livings are discussed in [4].

8. Modeling Virtual Life of L-particle

This topic is very specific for a L-particle, since for a physical particle it does not make sense.

    a. Life/non-life transformation. Let us turn to the feedback Eq. (10) and assume that

$$\zeta = 0 \text{ at } t > 0 \tag{54}$$

Then the feedback (10) vanishes, and the L-particle becomes a Newtonian one:

$$\dot{v} = 0$$

$$\rho = \rho_0 \tag{55}$$

Here the first equation represents the second Newton's law, and the second equation – the conservation of the probability density (the Liouville theorem). Obviously these equations are uncoupled as it is supposed to be in the Newtonian physics.

b. Transition to virtual life. In this sub-section we will discuss the capacity of **mathematical formalism** that provides an extension of the proposed model to a new space with imaginary time where L-particles exhibit virtual motions such as dreams and memories. In order to demonstrate that, let us replace Eq. (54) by the following

$$\zeta = \zeta_0 \frac{\sqrt{T-t}}{\sqrt{|T-t|}} \tag{56}$$

where T is the period of active life of the L-particle.

Then at $0 < t < T$

$$\zeta = 0 \tag{57}$$

the L-particle is "alive", and its activity is described by the governing equations (11) and (12).

For $t = T$

$$\zeta = 0$$

the L-particle "dies", and its Newtonian state is described by Eqs. (54) and (55).

But for $t > T$ the feedback (10)

$$f = i \frac{\zeta_0}{\rho(v,t)} \int_{-\infty}^{v} [\rho(\eta,t) - \rho^*(\eta)]d\eta \tag{58}$$

as well as Eqs. (11), (12) and (13)

$$\dot{v} = i \frac{\zeta_0}{\rho(v,t)} \int_{-\infty}^{v} [\rho(\eta,t) - \rho^*(\eta)]d\eta \tag{59}$$

$$\frac{\partial \rho}{\partial t} + i\zeta_0[\rho(t) - \rho^*] = 0 \tag{60}$$

become complex. For better interpretation, it will be more convenient to introduce an imaginary time

$$\tilde{t} = i\zeta_0 t \tag{61}$$

Then the formal solutions of these equations are

$$\rho = [(\rho_0 - \rho^*)e^{-\tilde{t}} + \rho^*] \tag{62}$$

$$\int_{-\infty}^{v} \rho^*(\eta)d\eta = (\frac{C}{e^{-\tilde{t}} - 1}), v \neq 0 \tag{63}$$

Thus the velocity v and the probability density ρ become real functions of imaginary time. It is reasonable to assume that the family of trajectories in the solution (63) describes virtual motions evolving in imaginary time with the probability (62), while the time scale of these motions could be different from the real one. Such a surrealistic activity can be associated with memories and dreams, and that is a unique property of livings.

Conclusion

The paper discusses a possible expansion of modern physics to include physics of Life since all attempts to create livings from non-living matter failed. Prior to the discussion, a life/non-life criterion is proposed: unlike physical systems, Livings can move from disorder to order without an external interference if the model of life concentrates upon the concept of **_intelligent behavior_** and does not include such "auxiliary" processes as metabolism and reproduction. A **_mathematical formalism_** suggests that a hypothetical "particle of life" (L-particle) can be represented by a quantum-classical hybrid in which the force following from the quantum potential is replaced by the **_information_** _force_. Besides the capability to move against the second law of thermodynamics, L-particle acquires such properties like self-image, self-awareness, self-supervision, est., that are typical for Livings. However since the L-particle being a quantum-classical hybrid acquires non-Newtonian and non-quantum properties, it does not belong to the physics matter as we know it: the modern physics should be complemented with the concept of the _information force_ that represents a bridge between non-living and living matter. It has been concluded that quantum mechanics and mechanics of livings are sub-classes of a broader physical model, and the difference between them is due to different Liouville feedbacks, i.e. in different information forces. At this stage, L-particle is introduced as an **_abstract mathematical concept_** that is satisfied only to mathematical rules and assumptions**,** and its physical representation is still an open problem. There is another argument in favor of a non-Newtonian approach to modeling life and intelligence. As pointed out by Penrose [5], the Gödel's famous theorem has the clear implication that mathematical understanding cannot be reduced to a set of known and fully believed computational rules. That means that no knowable set of purely computational procedures could lead to a computer-control robot that possesses genuine mathematical understanding. In other words, such privileged properties of living systems as common sense, intuition, or consciousness are non-computable within the framework of classical models. That is why a fundamentally new physics is needed to capture these "mysterious" aspects of livings, and the proposed mathematical abstraction could serve as a hint for experimental discoveries.

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