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Condensed Geometry: Quantum Canonical Phase Transitions in the Loop Quantum Gravity Regime

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Abstract: It was argued in the previous paper [1] that the Big Bang need not be singular in any theoretical framework. In the current paper, the new variables of Ashtekar type but more obvious from the point of view of quantum nature of time are used and exploited to study the phases of quantum geometry which in the author's new formulation [1] may be viewed as a fluid of surface states (or light cones). These phase transitions are discussed only after the quantum as well as the classical nature of time is established. The principle of equivalence - the bedrock of Einstein's general relativity, is shown to be a direct consequence of the lambda phase transition in a disordered phase of fluidic surface states; in the formalism of Loop Quantum Gravity. In short, this work deals with phase transitions in the Loop Quantum gravity or canonical general relativity regime.

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It was shown in the previous work [1] by the author that causality is the gauge degree of freedom of gravity and it is the intent of this paper to establish that the topologically connected fluid of surface states (or light cones centered at the nodes of the spin networks [4]) also has phase transitions which when interpreted cosmologically yields a dynamic universe. It is also established that the principle of equivalence is a direct consequence of the non-abelian nature of gravity.It is shown that these transitions are possible for a gas of surface states. The aim of the present paper is to present a different aspect of time and show how space time emerges from this new scenario and thereafter study the aspects of gravity as a collective phenomenon by means of condensed geometric phase

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transitions based on statistical mechanics and statistical thermodynamics of gravity [5] as a quantum system. This paper runs on par with the theory of Loop Quantum Gravity.

1. Incorporation of Time in Quantum Theory

Consider the configuration space C_{GR} of General Relativity whose cotangent bundle is the Hamiltonian phase space of the $SL(2,\mathbb{C})$ Yang-Mills gravity gauge. Here, for every ground state variable in the abstract vector space or the Dirac space, we have a wave function in the (Hilbert space) diffeomorphism invariant square integrable cross-sections of the vector bundle associated with the quantum system, here gravity - a system with finite number of degrees of freedom. Now, we have a simple familiar wavefunction $\psi(X) = \langle X|\psi\rangle = \exp\{i(pq - Et)\}$ for any quantum system embedded in gravity. For our purpose for this quantum system embedded in gravity, postulate the existence of an azimuthal 1-form - a bra:$\langle\varphi|$ whose inner product with the corresponding ket $|\psi\rangle$ yields the corresponding angular wave-function or spin function $\psi(\varphi) = \langle\varphi|\psi\rangle = \exp\{i(J_z\varphi - m\Gamma)\}$. Now this form we postulate as the equivalent form for the quantum mechanical wave function. We further postulate that every field phenomenon can be modeled into a spin system (see [8] and references therein) and thereby viewed as a collective phenomenon. This means that gravity also may be viewed as an Ising model of light cones centered at nodes

For our purpose we will consider the constraint surface of our quantum mechanical matter system imbedded in constraint surface of the gravity in the Hamiltonian phase space as it is the Hamiltonian constraint that generates the dynamics. Fix now an isomorphism, v, between the frame relative to which the quantum mechanical matter system is stationary and that of gravity i.e., the frame of the stationary observer in which the quantum system is dynamical. This isomorphism is the relative or Lorentzian velocity of the quantum system relative to the inertial frame of reference of our observer. The isomorphism allows for the velocities of both, the system's frame and the inertial frame, to be converted into each other via Einstein's velocity addition theorem. This velocity v can also be taken as the generalized velocity in the configuration space of general relativity under certain special circumstances. Incidentally this dynamics is generated by the action of the Hamiltonian constraint. This provides for a time independent Hamiltonian formulation of the Einstein velocity addition theorem. The relativistic nature of the dynamics is therefore conserved in the Poisson algebra of the Lagrange-Hamilton isomorphisms taken above. The constraint functionals generate canonical transformations which result in the rotation of the isomorphism indices. The isomorphisms are time-void generators of dynamics in 3-space. This is how the dynamics can be measured in the temporal gauge. The geodesic of the quantum system is coarse graining/ classical limit of the eigenvalue of the Thiemann length operator and moves causally in classical continuum. As the quantum system moves with a relative velocity v and thereby traces a geodesic in the continuum, the generalized velocity of this quantum system traces a trajectory on the constraint surface. Thus the transition of the quantum system from a state, $|\psi(0)\rangle = |n, R(0)\rangle\,, to another state, |\psi(t)\rangle = |n, R(t)\rangle$ is given by the transition amplitude expressed as the time-void path integral between two points on the constraint surface (the flows in time are in any case generated by the Hamilton constraint).$^+$

Define a vorticity, w of v, in a time-void $d = 3$ space - a fluid of future light cones, as

$$w := dv \tag{1}$$

where, the differential form convention has been used as in [1]. Now, fix an automorphism, u , of the constraint surface of gravity representing the bulk of the surface states/light cone fluid giving the collective dynamical behavior of the space - its velocity of flow. In other words, fix an isomorphism between two quantum fundamental observers' frames. This velocity also is time-void. For canonical vorticity, W, we have

$$\Gamma := \int_{\Sigma} w \cdot dS = \oint_{\lambda} v \cdot dl \tag{2}$$

where, Γ, the circuit integral is the circulation which is also time void.

The analytical continuity of the functions $\psi(x)$ and $\psi(\varphi)$ tells us that pq corresponds to $J_z\varphi$ and similarly Et corresponds to $m\Gamma$ so that we have by the Einstein theorem of inertia of energy that

$$Et = \frac{E}{c^2}\Gamma \tag{3}$$

Thus time arises naturally as a kind of vorticity in the temporally gauged Hamiltonian quantum dynamics, although gravity doesn't arise until curvature is involved; or

$$t = \frac{\Gamma}{c^2}. \tag{4}$$

Now, for an inviscid liquid, with canonical time ϑ measuring its motion in the configuration space of general relativity, the vorticity, w, satisfies the equation

$$\frac{\partial w}{\partial \vartheta} = curl\ u \wedge w. \tag{5}$$

Eq (5) tells us that the vortex lines are dragged with the fluid moving a velocity u. Thus, we perceive space-time rather than space and time. The Landau-Raychaudhuri equation tells us that vorticity causes expansion and shear causes contraction in space-time.

• Thus the time being dynamical vorticity, and since it is carried with space, causes expansion of the space-time and hence of the universe such that the rate of expansion is proportional to the 4-volume of the universe. By this, we have

$$\frac{dv}{dt} = K^4 V. \tag{6}$$

• By inserting the Planck scale parameters, we find that the rate constant is approximately equal to $6.161906892 \times 10^{246} cm^{-3} s^{-2}$. Thus there had to be an inflation of superluminal proportions and its aftermath would be an accelerating expanding universe.

It is the function of time to keep things moving in a particular direction in space-time or just in time. Progression is thereby the phenomenon of the nature and the expansion of space-time/universe is nothing but the progression of the entire space or space-time in time. We have also learnt a small physical moral:

• that vorticity aspect of dynamics of a system destroys the physical singularities occurring in the system. Here, the system is space-time and hence space-time singularities do not exist.

2. Canonical Phase Transitions: Emergence of Space-time and a Primer on the Hamiltonian Dynamics of Polyakov Strings.

Consider a gas of grannulons - the surface states of quantum geometry formed by the intersecting of the edges of the spin networks across an arbitrary surface. By a gas of surface states, we mean the spin networks being loosely interwined and not tight, for the energy is much higher than the Planck energy. Let $E \geq E_{Pl}$ be the energy of the gas of grannulons. The temperature of the ensemble is $T_G \geq T_{Pl}$ defined by

$$T_G = \left(\frac{\partial \sigma}{\partial E}\right)^{-1} \tag{7}$$

where σ is the entropy.

Consider the variation of the temperature of the gas. If the temperature T_G, of the gas of grannulons is decreased, the gas condenses to form a causal liquid. The partition function of this gas is

$$z = Tr\ \exp -\beta(E_r - \mu N_r), \tag{8}$$

where μ is the chemical potential of the grannulons. In the coarse graining limit, gravity or curved space-time is equivalent to a medium of refractive index greater than unity. It thus affects light by bending it. Now, canonically conjugate to the average energy of the grannulons is the earlier proved Lorentzian vorticity or Lorentzian time. The order parameter is by its very nature irreversible and so is time. So we choose for obvious reasons, the Lorentzian but global time-like parameter as the order parameter for our phase transitional alternative for singular Big Bang, also since vorticity leads to expansion, an expanding scenario alternative to the Big Bang Singularity (BBS) is possible. The partition function (8) changes in response to a change in entropy as

$$\sqrt{det\mathbf{P}}t = -\frac{\partial}{\partial E} lnz. \tag{9}$$

The quantity "$\det \mathbf{P}''$ is the determinant of the "momentum" 2-form $\mathbf{P}$ canonically conjugate to the Ashtekar connection triad, $A : A^i_a := \Gamma^i_a + ik^i_a$, on a 3-manifold M where Γ^i_a is the spin connection and k^i_a is the triad on the 3-surface parameterized by constant time coordinate. The Hamiltonian phase space Π consists of these canonically conjugate pairs $(A, \mathbf{P})$ both of which take values in the Lie-algebra $su(2)$. A discontinuity in the action

$$S = \left(T\frac{\partial P}{\partial T} - P\right) \tag{10}$$

can be expressed by the gap in the entropy σ[3],

$$disc\sigma = T_c disc\frac{\partial P}{\partial T} \tag{11}$$

From (9) with $lnz = \sqrt{\det \mathbf{P}}(P - S_0)$ where P is the thermodynamic pressure and S_0 is the ground state or pre-geometric action.(Henceforth, we consider the Thiemann trick in the quantum limit and therefore replace det$\mathbf{P}$ by the volume operator and for the large collection consider it to be of a finite expectation value when operated on a spin network wavefunction and dissolve into the action making action resulting into action density.) To this corresponds $\langle 0|t|0\rangle$; it follows that

$$t = \frac{\partial S_0}{\partial E} - \frac{\partial P}{\partial E} = \langle 0|t|0\rangle - \frac{\partial P}{\partial E}. \tag{12}$$

If we expand the pressure in the vicinity of T_c according to

$$P = P_c + (T - T_c)\frac{\partial P}{\partial T}|T_c + \cdots . \tag{13}$$

P depends on E via P_c and T_c

$$\frac{\partial P}{\partial E} = \frac{\partial P_c}{\partial E} - \frac{\partial T_c}{\partial E}\frac{\partial P}{\partial T}|_{T=T_c} \tag{14}$$

Inserting (14) into (12) and applying *"disc"* to both the sides, the result is

$$\sqrt{\det P}disct = \frac{\partial T_c}{\partial E}disc\frac{\partial P}{\partial T}|_{T=T_c}. \tag{15}$$

While P is discontinuous at T_c (as in the case of a first order phase transition), $\frac{\partial P}{\partial T}$ may jump. From (11) and (15), we finally obtain

$$\frac{\partial T_c}{\partial E} = T_c\sqrt{\det P}\frac{disct}{discS}. \tag{16 - a}$$

Since Lorentzian time as an order parameter goes vortical and thereby orders all dynamics by generating contact transformations in the phase space of pure gravity, equation (16-a) defines a canonically chiral condensate which is determined as a response of the partition function to a change in energy E of the gas as given in eq (9). The critical question is whether eq (16-a) leads to a bound on the latent heat in the first-order temporally chiral transition. What is the gas-liquid transition? I advocate that it is primordial/ pre-Planckian space time. The vapor of geometric particles condense to become a liquid which was the texture of the early universe spacetime. Whereas some geometric particles (grannulons) condensed to a liquid phase, some in the liquid phase escape into the gaseous phase. This escaping tendency is the fugacity of the thermogravity system. When this tendency is equal for both the phases, a thermodynamic equilibrium is said to have been reached. In eq (16-a), average energy of the gas of grannulons plays the role of fugacity. We propose a topological phase transition in the form of the Kosterlitz-Thouless phase transition which liberates the Lorentzian vortices and in the process breaks the $X - Y$

symmetry or the U(1) symmetry of the Ising Model of light cones or alternately geometric molecules - the nodes of the spin networks to which the light cones are attached [4] and thereby triggers the time which is the Lorentzian vortex and which being the order parameter, ticks off the first order phase transition almost simultaneously with the second disordered phase of gravity. This is the normal (component of) fluid and we proceed to derive the phase transition which yields the superconducting/superfluid part of this fluid. The universe materializes as an emergent phenomenon instead of a Big Bang scenario. The $SU(2)$ Ising model of light cones on the lines of the Markopoulou-Smolin causal evolution of spin networks will be solved exactly and the phase transitions discussed therein in another paper, perhaps, a sequel? For now, the current paper aims only at giving a simplistic model for phase transitions in non-perturbative canonical gravity, here, loop quantum gravity.

Now for a quantum string, more on the lines of Polyakov rather than the Nambu-Goto approach adopted by Thiemann (see [6] and references therein), which is shown to be marred with anomalies by a simple consideration of the harmonic oscillator by Helling and Policastro [7], we can show by arguments similar to the above that the dynamics of the string in space-time is a first order phase transition given by

$$\frac{\partial T_c}{\partial p^\mu} = \sqrt{\det \mathbf{P}} T_c \frac{discX^\mu}{discSPolyakov}. \tag{16 - b}$$

where p^μ is the momentum of the string conjugate to the world sheet coordinate X^μ. Thus, the quantum string regulates through the body of space-time just as a thread regelates through the bulk of the ice cube when pressed through it. The world-sheet momentum of the string is analogous to the pressure or rather fugacity in the usual Clausius-Clapeyron equation while X^μ is the order parameter. This is the subject of another paper.

3. Nucleation of Black Hole and its "Unruh Extension" to the Principle of Equivalence

We now have a back ground space-time with a global curvature. This we call the normal component of the two-fluid model. This, uptill the second transition temperature, is the gravity/quantum Riemannian geometry pervading "fluid". We therefore proceed to derive, the phase transition that leads to a black hole nucleating in a background spacetime- a second order phase transition in a grannulonic fluid. In our $SU(2)$ causality gauge theory [1], we are concerned with the generators of rotations, and the dynamical observable here is the intrinsic spin angular momentum of the black hole, which makes its appearance here in the change in the internal energy of the black hole, as

$$dU = \frac{\kappa c^2}{8\pi G} dA + \Omega dJ + \phi dQ. \tag{17}$$

We therefore by thermodynamical arguments choose the intrinsic angular momentum J as the order parameter and build the gravitational Gibbs' function as

$$G = U - frac\kappa c^2 8\pi G A - \Omega J - \phi Q, \tag{18}$$

so that,

$$dG = -Ad\kappa \frac{c^2}{8\pi G} - Jd\Omega - Qd\phi. \tag{19}$$

$$\left(\frac{\partial G}{\partial \kappa}\right)_{\Omega,\phi} = \frac{c^2}{8\pi G}A, \tag{20 - a}$$

$$\left(\frac{\partial G}{\partial \Omega}\right)_{\kappa,Q} = -J, \tag{20 - b}$$

$$\left(\frac{\partial G}{\partial \phi}\right)_{\kappa,\Omega} = -Q. \tag{20 - c}$$

$$\left(\frac{\partial \phi_c}{\partial \kappa}\right)_{\Omega} = \frac{\left(\frac{\partial G_n}{\partial \kappa}\right)_{\Omega,\phi} - \left(\frac{\partial G_s}{\partial \kappa}\right)_{\Omega,\phi}}{\left(\frac{\partial G_n}{\partial \phi}\right)_{\Omega,\kappa} - \left(\frac{\partial G_s}{\partial \phi}\right)_{\Omega,\kappa}} \tag{(21}$$

where the subscripts "n" and "s" stand for the normal and the superconducting components of the two component fluid that we are considering. We therefore have,

$$\left(\frac{\partial \phi_c}{\partial \kappa}\right)_{\Omega} = -\frac{c^2}{8\pi G}\frac{A_n - A_s}{Q_n - Q_s}. \tag{(22}$$

Similarly,

$$\left(\frac{\partial \phi_c}{\partial \Omega}\right)_{\kappa} = -\frac{J_n - J_s}{Q_n - Q_s}. \tag{23}$$

Or therefore,

$$\left(\frac{\partial \kappa}{\partial \Omega}\right)_{\phi_c} = \frac{8\pi G}{c^2}\frac{J_n - J_s}{A_n - A_s}. \tag{24}$$

which is identical with the Clausius-Clapeyron equation. For (16-a) describing a $SU(2)$ locally Lorentzian space-time geometry, eq (24) shows the locally Lorentzian spacetime geometry to transit to a non-abelian superconducting state so that all the Ashtekar "electric" flux tubes get pushed into the "bags" of large energy momentum such as black holes. In fact globally, we argue based on the work of Unruh [2] that all energy-momentum distributions are in fact energy-momentum bags. In general eq (24) may be written as

$$\left(\frac{\partial a}{\partial \Omega}\right)_{\phi_c} = \frac{8\pi G}{c^2}\frac{J_n - J_s}{A_n - A_s}. \tag{25}$$

This equation is the essence of both, the weak and the strong principle of equivalence. The principle of equivalence is thus a connectioelectric Meissner effect. This superconducting phase is characterized by the existence of locally Lorentzian space-time domains wherein there exist gravitational/ Galilean supercurrents characterized by rectilinear geodesics or inertial frames. This interpretation may be digressed as follows: in an electric conductor at normal temperatures, the electric current is hindered by the resistance of the medium of the conductor. In the super conducting phase, the resistance drops to zero and the supercurrents ensue. The electrons thus, travel more swiftly from one point to another. Similarly, the presence of a curvature is a hindrance to the motion of a test object. When spacetime is locally Lorentzian i.e., free of gravity, it allows the object to travel swiftly from one point to another along a straight line. Thus, special relativity is a direct and natural consequence of Galilean supercurrents. The non-abelian nature of the gravity gauge field yields the weak and the strong principle of equivalence which state that

Weak Equivalence Principle: Effects of gravitation can be transformed away locally and over small intervals of time by using suitably accelerated frames of reference.

Strong Equivalence Principle: Any non-gravitational physical interaction behaves in a locally inertial frame as if gravitation were absent.

• The interior of the black hole or an energy-momentum bag is that of perturbative vacuum whereas that of the empty space is the normal or non-perturbative vacuum. This adds to the possibility of engineering of the vacuum.

4. Conclusions

Overall an alternative to the singular Big Bang cosmology is successfully constructed along with a new vortical aspect of Lorentzian time as well as global time-like parameters considered in the quantum dynamics of loop quantum gravity. Einstein's pure gravity starts from the empirical principle of equivalence and arrives at curved space-time by bending and orienting the future light cones. Our theory starts with a fluid of surface states (or light cones) and being a statistical quantum theory of gravity, investigates phase transitions that produces the emergent chiral gravity/spacetime and the connectio-electric Meissner effect and thereby renders the blackholes and principle of equivalence. The heavy energy-momentum bags orient domains of $SU(2)$ dipoles viz., the future light cones, close to them as in the case of black hole wherein the future dipoles are trapped within the horizons. This is the $SU(2)$ dual of the U(1) ferromagnetic phase representing strong gravity just as ferromagnetism is the highest degree of magnetic response manifested by the $U(1)$ gauge. At this juncture, we are in a position to formulate a conjecture as a result of our investigations:

• Lorentzian time is a background free vorticity arising from Hamiltonian classical or quantum dynamics.

• Every d- dimensional non-perturbative gauge theoretic phenomenon has a $d+1$ dimensional perturbative gravity dual. In fact, we have from this an alternate formulation for the latter part of our conjecture:

• Every d-dimensional perturbative vacuum of gauge theory has d+1 dimensional perturbative gravity dual.

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